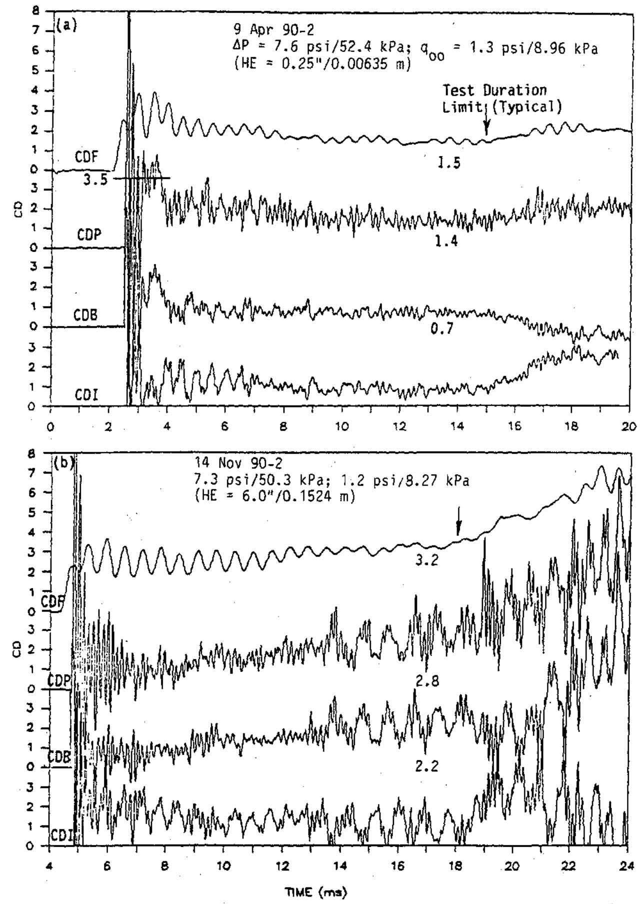

Fig. 2. Typical CD coefficient time histories ($\Delta P \approx 7.5$ psi/51.7 kPa; clean; without berm).

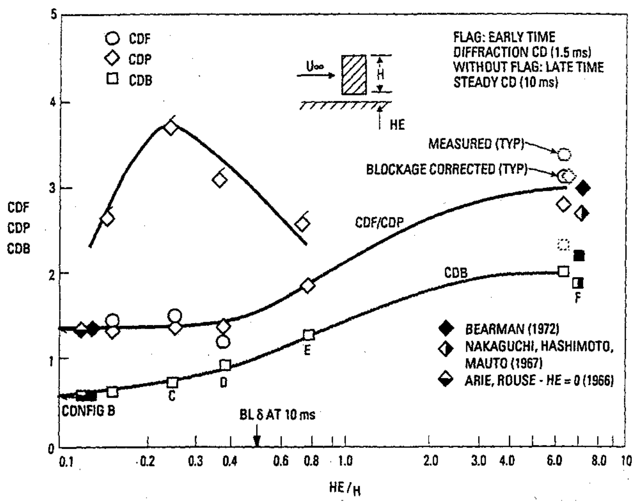

Fig. 3. Dependence of drag coefficient on elevation ($\Delta P \approx 7.5$ psi/51.7 kPa; clean; without berm).

critical to loads evaluation studies as fineness ratio, Reynolds number, and corner rounding considerations. Note the favorable comparison between the current steady-state data for the two elevation limit cases, $HE \to 0$ and $HE \to \infty$, with earlier results.[3,9,10] Apparently the influence of boundary-layer buildup ($\delta_{10\,ms} \approx 0.5$ in./0.0127 m) on measured CD results is "small" as demonstrated by the favorable agreement of the low *HE* data with established splitter-plate results. Also, the effects of vortex shedding (evidenced by the presence or absence of periodicity in the CDB data), as already discussed and reviewed by Bearman[5] and Vickery,[11] can be seen to play a major role in maintaining low base pressures (high CDB) at high elevation ($HE/H = 6$, Fig. 2b) and low CDB at low elevation where base scavenging due to vortex shedding is substantially reduced ($HE/H = 0.25$, Fig. 2a).

Because of the sensitivity of base pressure to the vortex shedding phenomena, investigators have pointed out that reduction in base drag can be achieved by modest geometric changes that interfere with the vortex generating mechanism. Such configuration modifications include fineness ratio,[3] corner rounding,[2] splitter-plate length,[3-5] trailing-edge spoiler,[3] and surface proximity (current results). Note, for example, the onset at late time of periodicity in the CDB signal at $HE/H = 6$ in Fig. 2b (the vortex shedding "signature") and the absence of any periodicity for the data for $HE/H = 0.25$ (e.g., Fig. 2a). Since the drag "beat" frequency is typically twice that of the corner shedding frequency,[10] an estimate can be made as to the Strouhal number (S) for the current study

$$S = \bar{n}H/u_\infty$$

where $\bar{n}$ represents the Strouhal frequency ($\approx 0.5 \times$ drag periodicity), H the model height, and u_∞ the local freestream velocity. The computed value so derived is seen to be somewhat higher (0.15) than previous measurements (0.13) (Ref. 3). The onset of CDB periodicity is also consistent with the observed increase in base drag as would be expected since base scavenging is controlled by the establishment of the von Kármán vortex street.[3,4]

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Fluid Column Stability in the Presence of Periodic Accelerations

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Introduction

THE float zone configuration is used in crystal growth. It may be modeled as a liquid column held by surface ten-

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sion forces between two end disks. In the case of crystal growth, the thermal and solutal fields as well as those of velocity and pressure are needed to characterize the physics. A compelling reason for crystal growth experiments in a microgravity environment such as that onboard the Space Shuttle is that buoyancy forces are greatly reduced.[1] However, this environment is not quiescent due to the presence of impulse type disturbances from small thruster firings as well as those from periodic vibrations. Given certain "environmental" conditions, such as the presence of a periodic acceleration field, it is possible that a fluid dynamical instability would develop. This would adversely effect the crystal growth.

The crystal growth environment cannot be separated from the fluid dynamics of the liquid column. This latter topic is the focus of the present work. The question of column interface stability in the presence of a periodic acceleration field having a component normal to the longitudinal axis of the isothermal cylinder is investigated. The fluid column is taken to be infinite in length. Floquet theory is used in the stability investigation.

Previous work has determined the natural oscillations of the liquid column. This work was done first for the case of the infinite column[2] and later for the finite length case. In the latter work, both axisymmetric and nonaxisymmetric oscillations were considered.[3,4] Results for the infinite length case were found to be good approximations to those for the finite length column, both numerically and with regard to trends.

Interface behavior of the finite length liquid column in the presence of time-dependent forcing has been investigated for the case in which the forcing was parallel to the longitudinal axis of the column.[5,6] The investigations considered the column behavior subject to a sin(t) forcing for both the inviscid case of general aspect ratio (within static stability limits[5]) and the viscous case in the slender column limit.[6]

The problem of interface stability of the fluid column in the presence of a periodic acceleration field that has a component normal to the longitudinal axis of the column has not been investigated. Previous work has considered the interface stability of a highly idealized infinite slab-like configuration in the presence of a periodic acceleration field oriented normal to the interface.[7] Interestingly, this study was motivated by experiments in microgravity.

Use of the infinite length configuration in this stability study results in a simplification in that the standard boundary conditions at the solid end disks are not applied, and the focus remains on the interface stability. If an extension of this work to the finite length configuration is of interest, Floquet analysis would be appropriate, although the implementation would be more complicated.

Formulation

The basic configuration is that of an infinite fluid column of circular cross section. The fluid is incompressible, and the surrounding medium is of negligible density. Perturbations are taken to be irrotational. The analysis is linear and inviscid, with the nondimensionalized governing equations those of continuity and conservation of momentum (linearized Euler).

The frequency of the periodic forcing is denoted by ω_f. Pressure and velocity fields are given by p and $\boldsymbol{u}$, nondimensionalized as follows:

$$R\tilde{x} = x \qquad \omega_f^{-1}\tilde{t} = t \qquad R\omega_f\tilde{u} = \boldsymbol{u} \qquad \rho(R\omega_f)^2\tilde{p} = p \tag{1}$$

Tildes indicate nondimensional quantities.

The continuity and Euler equations are then

$$\tilde{\nabla}\cdot\tilde{u} = 0 \tag{2a}$$

$$\frac{\partial\tilde{u}}{\partial\tilde{t}} + \tilde{u}\cdot\tilde{\nabla}\tilde{u} = -\tilde{\nabla}\tilde{p} - Fr\cos(\tilde{t})\tilde{\nabla}[(1-\tilde{r})\sin\theta] \tag{2b}$$

with $Fr = (Go/R\omega_f^2)$ a Froude type number. Go is the amplitude of the periodic acceleration field. The functional form of the time-dependent forcing is selected to be cos($\tilde{t}$).

The mean state is one of zero velocity; however, the mean pressure is time dependent. Consider a wave-like perturbation propagating on the interface. The governing equations for the perturbation are developed as follows. Expand in the small perturbation parameter ϵ

$$\tilde{p} = p_{m(\mathrm{ean})} + \epsilon p \qquad \tilde{u} = \epsilon\boldsymbol{u} = \epsilon\nabla\phi \tag{3}$$

Substitution of Eq. (3) into Eqs. (2) yields the mean system

$$\nabla p_{m(\mathrm{ean})} = -Fr\cos(t)\nabla[(1-r)\sin\theta] \tag{4}$$

(with the parameter Fr of order one) and the (order ϵ) perturbation equations

$$\nabla^2\phi = 0 \tag{5a}$$

$$\frac{\partial(\nabla\phi)}{\partial t} = -\nabla p \tag{5b}$$

The spatial dependence of the velocity potential can be determined via solution of Eq. (5a), which is

$$\phi(r,\theta,z,t) = \Sigma A(t)I_m(kr)\exp(ikz)\exp(im\theta) \tag{6}$$

I_m is the mth modified Bessel function. Clearly, the solution involves a superposition of the azimuthal modes.

It is through the boundary conditions that the free interface can be determined and the stability characteristics investigated. Let the equilibrium interface be given by

$$Fe = r - 1 - \epsilon\eta(\theta,z,t) = r - 1 - \epsilon\Sigma C(t)\exp(ikz + im\theta) \tag{7}$$

The kinematic condition and the normal force balance at the free interface must be satisfied. In addition, the requirement of conservation of mass, which reduces to a conservation of volume condition, must hold.

The linearized kinematic condition (at order ϵ) is

$$-\frac{\partial\eta}{\partial t} + u_r = 0 \tag{8a}$$

at $r = 1$ with $u_r = \partial\phi/\partial r$. This results in

$$\Sigma\left(\frac{\mathrm{d}C}{\mathrm{d}t}\right)\exp(ikz + im\theta) = \Sigma[I_m(k)]'A(t)\exp(ikz + im\theta) \tag{8b}$$

The normal force balance requires that the difference in pressure across the interface be equal to the curvature multiplied by the surface tension force. In nondimensional form, this is given by

$$Bo\times\Delta(p_{m(\mathrm{ean})} + \epsilon p) = \nabla\cdot\mathbf{n} \tag{9a}$$

with $\mathbf{n} = \nabla Fe/\|\nabla Fe\|$. Bo is the nondimensional parameter $(\rho R^3\omega_f^2/\gamma)$, with γ the surface tension and ρ the density. At order ϵ, the linearized form of this interfacial condition can be expressed as

$$\eta + \eta_{zz} + \eta_{\theta\theta} + [Bo\cos(t)\sin\theta]\eta = Bo\left(\frac{\partial\phi}{\partial t}\right) \tag{9b}$$

Fr, required to be of order one, has been set equal to unity. The subscripts indicate partial differentiation. This yields

$$\Sigma\{(1 - m^2 - k^2) + Bo\cos(t)\sin\theta\}\times C(t)\exp(ikz + im\theta)$$
$$= \Sigma A(t)I_m(k)\exp(ikz + im\theta) \tag{9c}$$

The sin θ dependence can be re-expressed as an exponential function. Then

$$C_m(t)(1 - m^2 - k^2) + \left(\frac{Bo}{2}\right)(-i)\cos(t)[C_{m-1}(t)$$
$$- C_{m+1}(t)] = BoI_m(k)\left(\frac{\mathrm{d}A_m}{\mathrm{d}t}\right) \tag{9d}$$

and azimuthal mode coupling occurs. Use of Eqs. (8b) and (9d) yields a nonautonomous second-order equation in $C_m(t)$, with mode coupling (as indicated by the subscripts m)

$$C_m'' - [(1 - m^2 - k^2)/Bo]\{I_m'/I_m\}C_m = \{I_m'/2I_m\}\cos(t)(-i)[C_{m-1}(t) - C_{m+1}(t)] \qquad (10)$$

Keep in mind that $\cos(t)$ can be rewritten in exponential form. It is at this juncture that Floquet analysis is applied. For convenience, let $E_m(t) = [\mathrm{d}C_m(t)/\mathrm{d}t]$. Then take

$$[C_m(t), E_m(t)] = \sum_{l=-\infty}^{l=\infty}[C_{m,l}, E_{m,l}] \times \exp[(\lambda + il)t] \qquad (11)$$

The constant coefficients $C_{m,l}$ and $E_{m,l}$ are unknown. The Floquet exponent is denoted by the eigenvalue λ, which is in general a complex number, and which is also unknown. The nonautonomous differential system is thus transformed into a homogeneous algebraic system for $(C_{m,l}, E_{m,l})$ with the unknown parameter (eigenvalue) λ. If Real(λ) is greater than zero, the interface of the cylindrical column is unstable to the growing wave-like perturbation. Of course, the milieu in which this disturbance is propagating includes the periodic base state pressure.

Use of Eq. (11) in Eq. (10) (rewritten as two first-order modes) yields the infinite algebraic system given by

$$(\lambda + il)C_{m,l} = E_{m,l} \qquad (12a)$$

$$(\lambda + il)E_{m,l} = [(1 - m^2 - k^2)/(Bo)](I_m'/I_m)C_{m,l} + (\tfrac{1}{4}I_m'/I_m)(-i)(C_{m-1,l-1} + C_{m-1,l+1} - C_{m+1,l-1} - C_{m+1,l+1}) \qquad (12b)$$

Note that the harmonic modes (indicated by l) as well as the azimuthal modes (indicated by m) are coupled to both their preceding and successive modes.

Several remarks are in order concerning the truncation. Once the truncation in m is done, the number of azimuthal modes that contribute are fixed. It is to this truncated system that Floquet analysis is actually applied. To obtain numerical values for λ, it is necessary to truncate the number of harmonic modes in time, i.e., the range of l values. The eigenvalue problem is, therefore, a problem of the truncated system.

Results

The results pertain to the eigenvalue solutions of system (12a) and (12b). NAG library routines were used in determining the eigenvalues. Truncation values of $L = |15|$, that is, $-15 \le l \le 15$, and $M = 14$ were found to be sufficient. Wave number values ranged from $k = 0.10$ to $k = 3.00$. The parameter Bo was varied from 0.01 to 10.00.

For $k < 1.0$, the interface is unstable to the wave-like perturbation in the presence of a mean periodic acceleration field over the range of Bo values considered. (Note that this range encompasses four orders of magnitude; see Fig. 1.)

As k is increased, such that $1.00 \le k \le 1.20$, the interface remains unstable to the perturbation for the two larger Bo values, equal to 10.00 and 1.00. However, marginal stability [Real(λ) = 0] ensues at the smaller Bo values. A decrease in Bo can be interpreted physically as an increase in the surface tension. So, as the surface tension increases, the restoring force is sufficient to result in marginal stability over this range of wave numbers.

Perturbations corresponding to the larger wave numbers (smaller wavelengths) that were considered, $2.0 \le k \le 3.0$, were found not to grow in time for Bo = 0.01–1.00. That is, instability [with Real(λ) > 0] of the interface to perturbations of these larger wave numbers (and smaller wavelengths) occurs only for Bo = 10.00; otherwise, Real(λ) = 0. An alternative physical interpretation to that involving a variation in surface tension for differing Bo values can be developed. Since Fr was taken to be unity, ω_f^2 (the forcing frequency) is proportional to Go, the amplitude of the periodic acceleration field. Utilizing this relation in the definition for Bo yields $Bo \alpha (Go/\gamma)$. For fixed surface tension (and, of course, density and column radius) values, an increase in Bo would result from an increase in forcing amplitude. It is at the highest such amplitude considered that the interface was found to be unstable [with Real(λ) > 0] to the perturbation.

It is noted that the range of Bo values used corresponds to values of Go and ω_f that would be of interest in a microgravity environment for certain ranges of surface tension values. (Roughly, $10^{-4}g_{\text{earth}} \le Go \le 10^{-2}g_{\text{earth}}$, and 0.5 Hz $< \omega_f <$ 5 Hz for γ values of 1–100 dynes/cm.)

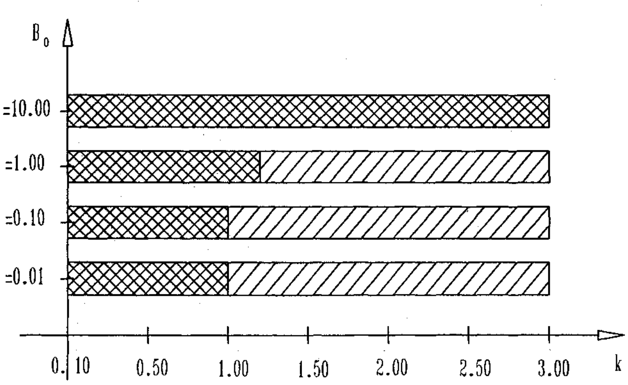

Fig. 1 Stability diagram: the stability of the configuration to wave-like perturbations for a range of *Bo* parameter values vs wave number *k* is shown. The cross-hatched area indicates unstable regions in which disturbances are growing in time. The remaining area corresponds to that of the marginal stability state.

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Coherent Structure Interactions in Excited Coaxial Jet of Mean Velocity Ratio of 0.3

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Nomenclature

D = outer diameter of outer nozzle
d = inner diameter of inner nozzle

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